INTRODUCTION TO NUCLEAR ENGINEERING **Homework 2.1**

Problem: An accelerator increases the total energy of electrons uniformly to 10 GeV over a 3000 m path. That means that at 30 m, 300 m, and 3000 m, the kinetic energy is 10^8, 10^9, and 10^{10} eV, respectively. At each of these distances, compute the velocity, relative to light (**v/c**), and the mass in atomic mass units.

Solution: The kinetic energy ***T*** is equal to the difference between the total energy (**mc^2**)and the particle's (i.e., the electron's) rest energy as given by Eq. (2.10), **$T = mc^2 - m_e c^2$**. Next, Eq. (2.2) can be rewritten to determine the velocity ***v*** relative to the speed of light ***c*** by solving for ***v/c***.

Evaluation at 30 m:

$E_{T30} := \dfrac{10^8}{10^6}$ MeV, Total energy of electron at 30 meters

$c := 2.99792458 \cdot 10^8$ m/s, Speed of light

$m_{30} := \dfrac{E_{T30}}{c^2} \cdot \dfrac{c^2}{931.494013}$ amu, Mass of electron at 30 m

$m_{30} = 0.1074$

$m_e := 5.48579911 \cdot 10^{-4}$ amu, Rest mass of an electron (Table 1.5)

$v\_c_{30} := \sqrt{1 - \left(\dfrac{m_e}{m_{30}}\right)^2}$ dimensionless, Ratio of v/c at 30 meters

$v\_c_{30} = 9.9998694392 \times 10^{-1}$

Evaluation at 300 m:

$E_{T300} := \dfrac{10^9}{10^6}$ MeV, Total energy of electron at 300 meters

$c := 2.99792458 \cdot 10^{8}$ m/s, Speed of light

$m_{300} := \frac{E_{T300}}{c^2} \cdot \frac{c^2}{931.494013}$ amu, Mass of electron at 300 m

$m_{300} = 1.0735$

$m_e := 5.48579911 \cdot 10^{-4}$ amu, Rest mass of an electron (Table 1.5)

$v\_c_{300} := \sqrt{1 - \left(\frac{m_e}{m_{300}}\right)^2}$ dimensionless, Ratio of v/c at 300 meters

$v\_c_{300} = 9.9999986944 \times 10^{-1}$

Evaluation at 3000 m:

$E_{T3000} := \frac{10^{10}}{10^{6}}$ MeV, Total energy of electron at 3000 meters

$c := 2.99792458 \cdot 10^{8}$ m/s, Speed of light

$m_{3000} := \frac{E_{T3000}}{c^2} \cdot \frac{c^2}{931.494013}$ amu, Mass of electron at 3000 m

$m_{3000} = 10.7354$

$m_e := 5.48579911 \cdot 10^{-4}$ amu, Rest mass of an electron (Table 1.5)

$v\_c_{3000} := \sqrt{1 - \left(\frac{m_e}{m_{3000}}\right)^2}$ dimensionless, Ratio of v/c at 30 meters

$v\_c_{3000} = 9.9999999869 \times 10^{-1}$

This concludes the problem solution for **Homework Problem 2.1**!

[HW_2_1.xmcd]

Problem: Consider a fast moving particle whose relativistic mass **m** is **100ε** percent greater than its rest mass m_o, i.e., $m = m_o(1+\varepsilon)$.

(a) Show that the particle's speed ***v***, relative to that of light is $v/c=(1-1/(1+\varepsilon^2))^{0.5}$

(b) For $v/c<<1$, show that this exact result reduces to v/c is approximately $(2\varepsilon)^{0.5}$

Solution: The solution using Eq. (2.2) is as follows when we let **vvcc = v/c**:

$$m = m_o\cdot(1+\varepsilon)$$

From the problem definition

$$m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$$

Relativistic mass with respect to the rest mass from Eq. (2.2)

$$m = \frac{m_o}{\sqrt{1-vvcc^2}}$$

$$m_o\cdot(1+\varepsilon) = \frac{m_o}{\left(1-vvcc^2\right)^{\frac{1}{2}}}$$

$$vvcc = \frac{\left(2\cdot\varepsilon+\varepsilon^2\right)^{\frac{1}{2}}}{1+\varepsilon} = \sqrt{\left[\frac{2\cdot\varepsilon+\varepsilon^2}{(1+\varepsilon)^2}\right]} = \sqrt{\left[\frac{2\cdot\varepsilon+\varepsilon^2+1-1}{(1+\varepsilon)^2}\right]} = \sqrt{\left[\frac{(1+\varepsilon)^2-1}{(1+\varepsilon)^2}\right]}$$

$$vvcc = \sqrt{1-\frac{1}{(1+\varepsilon)^2}}$$

Now if ***v/c*** is very small, then utilizing the result above and the Taylor Series expansion, Eq. (2.2) becomes:

$$\mathrm{vvcc} = \sqrt{1 - \frac{1}{(1+\varepsilon)^2}} = \sqrt{1 - \frac{1}{(1+2\cdot\varepsilon)}} = \sqrt{\frac{(1+2\cdot\varepsilon)}{(1+2\cdot\varepsilon)} - \frac{1}{(1+2\cdot\varepsilon)}} = \sqrt{\frac{(1+2\cdot\varepsilon-1)}{(1+2\cdot\varepsilon)}}$$

Note that 1+ε is approximately equal to 1 when ***v/c*** is very small!

$$\mathrm{vvcc} = \sqrt{\frac{2\cdot\varepsilon}{1}} = \sqrt{2\cdot\varepsilon}$$

This concludes the problem solution for **Homework Problem 2.2**!

[HW_2_2.xmcd]

Problem: In fission reactors, one deals with neutrons having kinetic energies as high as 10 MeV. How much error is incurred in computing the speed of 10 MeV neutrons by using the classical expression rather than the relativistic expression for kinetic energy?

Solution: We must determine ***v***, the velocity of the neutrons by both classical and relativistic expressions. The solution using Eq. (2.10) for the relativistic expression for kinetic energy is as follows:

$m_n := 1.0086649233$ amu, Rest mass of neutron

$T := 10$ MeV, Kinetic energy of neutrons

Classical Mechanics

$$T = \frac{1}{2} \cdot m_n \cdot v_{classical}^2$$

$$v_{classical} := \sqrt{\frac{2 \cdot T}{m_n}}$$ (MeV/amu)$^{1/2}$, Speed of neutrons in classical mechanics approach

$$v_{classical} = 4.4529$$

Relativistic Mechanics

$$T = m_{rel} \cdot c^2 - m_n \cdot c^2 = m_n \cdot c^2 \left(\sqrt{\frac{1}{1 - \frac{v_{rel}^2}{c^2}}} - 1 \right)$$

$$\frac{T}{\left(m_n \cdot c^2\right)} + 1 = \sqrt{\frac{1}{1 - \frac{v_{rel}^2}{c^2}}}$$

$c := \sqrt{931.49}$ (MeV/amu)$^{1/2}$, Energy-to-mass conversion factor for the speed of light

$$v_{rel} := \left(2\cdot T\cdot m_n\cdot c^2 + T^2\right)^{\frac{1}{2}}\cdot\frac{c}{T + m_n\cdot c^2}$$

(MeV/amu)$^{1/2}$, Speed of neutrons in relativistic mechanics approach

$$v_{rel} = 4.4177$$

$$v_{error} := \left(\frac{v_{classical}}{v_{rel}} - 1\right)\cdot 100$$

%error in using classical mechanics approach to calculate neutron velocity

$$v_{error} = 0.7965 \ \%$$

This concludes the problem solution for **Homework Problem 2.3**!

[HW_2_3.xmcd]

INTRODUCTION TO NUCLEAR ENGINEERING **Homework 2.4**

Problem: What speed (m/s) and kinetic energy (MeV) would a neutron have if its relativistic mass were 10% greater than its rest mass?

Solution: We will use some of the results from Problem 2.3 to calculate the relativistic speed and the kinetic energy of the neutron along with Eq. (2.10).

$m_n := 1.0086649233$ amu, Rest mass of neutron

$m_{rel} := 1.10 \cdot m_n$ amu, Relativistic mass of neutron

$c := \sqrt{931.49}$ $(MeV/amu)^{1/2}$, Energy-to-mass conversion factor for the speed of light

Relativistic Mechanics

$T := m_{rel} \cdot c^2 - m_n \cdot c^2$ MeV, Kinetic energy of neutron

$T = 93.9561$

$$T = m_{rel} \cdot c^2 - m_n \cdot c^2 = m_n \cdot c^2 \left(\sqrt{\frac{1}{1 - \frac{v_{rel}^2}{c^2}}} - 1 \right)$$

$$\frac{T}{\left(m_n \cdot c^2\right)} + 1 = \sqrt{\frac{1}{1 - \frac{v_{rel}^2}{c^2}}}$$

$$v_{rel} := \left(2 \cdot T \cdot m_n \cdot c^2 + T^2\right)^{\frac{1}{2}} \cdot \frac{c}{T + m_n \cdot c^2}$$

$(MeV/amu)^{1/2}$, Speed of neutrons in relativistic mechanics approach **(see results from Problem 2.3)**

$v_{rel} = 12.7147$

$$v_{rel} := v_{rel}\cdot\sqrt{\frac{\left(2.998\cdot10^{8}\right)^{2}}{931.49}}$$ m/s, Speed of neutron

$$v_{rel} = 1.2490\times10^{8}$$

It is interesting to note the conversion factor to get from **(MeV/amu)$^{1/2}$** to meters/second. Remember that the equivalent conversion for the square of the speed of light, **c^2**, is **931.49 MeV/amu.** Obviously, then to get the units for **c**, the square root of **931.49 MeV/amu** must be taken and remember that **c = 2.998*10^8 meters/second**! Always perform a units check to ensure your conversions are executed properly.

This concludes the problem solution for **Homework Problem 2.4**!

[HW_2_4.xmcd]

Problem: In a Relativistic Heavy Ion Collider, nuclei of gold are accelerated to speeds of 99.95% the speed of light. These nuclei are almost spherical when at rest; however, as they move past the experimenters they appear considerably flattened in the direction of motion because of relativistic effects. Calculate the apparent diameter of such a gold nucleus in its direction of motion relative to that perpendicular to the motion.

Solution: Fist we will write down what we know from the problem definition such as ***v/c*** and the diameter of a gold nucleus. We will then look at the change in length due to relativistic effects as it may apply to the diameter of the gold nucleus.

$c := 2.998 \cdot 10^{8}$ m/s, Speed of light

$fac := 0.9995$ dimensionless, Ratio of speed of gold nuclei to the speed of light

$v := fac \cdot c$ m/s, speed of gold nuclei

$A_{Au} := 196.96655$ amu, Atomic weight of gold

$R_{o} := 1.25 \cdot 10^{-13}$ cm, Spherical radius constant (see Eq. (1.7))

$R_{Au} := R_{o} \cdot A_{Au}^{\frac{1}{3}}$ cm, Calculated radius of gold nucleus per Eq. (1.7) of text

$D_{o} := 2 \cdot R_{Au}$ cm, Nucleus diameter as it appears when "stationary"

$D_{exp} := D_{o} \cdot \sqrt{1 - fac^{2}}$ cm, Length (or diameter) as it appears to experimenters watching the gold nuclei fly by

$D_{exp} = 4.5992 \times 10^{-14}$

This concludes the problem solution for **Homework Problem 2.5**!

[HW_2_5.xmcd]